COMMENTS OF BELLSOUTH

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ATTACHMENT 2, PART 1

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ATTACHMENT 2

SEEM PLAN

(Self-Effectuating Enforcement Mechanism)

Service Performance Measurements And Enforcement Mechanisms Administrative Plan

1 Scope

1.1 This Administrative Plan ("Plan") includes Service Quality Measurements ("SQM") with corresponding Self Effectuating Enforcement Mechanisms ("SEEM") applicable to this agreement.

1.2 All exhibits referred to in this attachment are located on the BellSouth Performance Measurement Reports website.

2 Reporting

2.1 In providing services pursuant to this Agreement, BellSouth will report its performance to the CLEC in accordance with BellSouth's SQMs and applicable SEEMs, which are posted on the Performance Measurement Reports website.

2.2 BellSouth will make performance reports available to the CLEC on a monthly basis. The reports will contain information collected in each performance category and will be available to the CLEC via the Performance Measurements Reports website. BellSouth will also provide electronic access to the raw data underlying the SQMs.

2.3 Preliminary SQM reports will be posted on the Performance Measurements Reports website by 8:00 A.M. EST on the 21st day of each month or the first business day after the 21st for the previous month's performance. Final validated SQM reports will be posted by 8:00 A.M. EST on the last day of the month. SQM reports not posted by this time will be considered late for SEEM purposes.

2.4 Final validated SEEM reports will be posted on the 15th of the month, following the final validated SQM report.

3 Modifications to Measurements

3.1 Service Quality Measurements

3.1.1 BellSouth will review the SQMs semi-annually. All modifications to the SQMs will be approved by Federal Communications Commission (the "Commission"). The CLEC may provide input to BellSouth regarding any suggested additions, deletions or other modifications to the SQMs. BellSouth will provide notice of all changes to the SQMs via the Performance Measurement Reports website.

3.1.2 Notwithstanding the foregoing, BellSouth may, from time to time, be ordered by a regulatory or judicial body to modify or amend the SQMs. BellSouth will make all such changes to the SQMs pursuant to any such orders.

3.1.3 Notwithstanding any other provision of this document, in the event a dispute arises regarding the modification or amendment of the SQMs, the parties will refer the dispute to the Commission.

3.2 Self Effectuating Enforcement Mechanisms and Statistical Test

3.2.1 In order for BellSouth to accurately administer Enforcement Mechanisms, the SEEMs shall be modified or amended only if BellSouth determines such modification or amendment is necessary. However, BellSouth will not delete any effective SEEM without prior written consent of the Commission. BellSouth will notify the affected parties of any such modification or amendment to the SEEMS in a timely manner.

3.2.2 Notwithstanding the foregoing, BellSouth may, from time to time, be ordered by a regulatory or judicial body to modify or amend then SEEMs and/or Statistical Test. BellSouth will make all such changes to the SEEMs and/or Statistical Test pursuant to any such order.

3.2.3 Notwithstanding any other provision of this document, in the event a dispute arises regarding the modification or amendment of the SEEMs and/or Statistical Test, the parties will refer the dispute to the Commission.

4 Enforcement Mechanisms

4.1 Definitions

4.1.1 Enforcement Measurement Elements means the performance measurements identified as SEEM measurements in the SQM.

4.1.2 Enforcement Measurement Benchmark means a competitive level of performance negotiated by BellSouth used to evaluate the performance

of BellSouth and the CLEC where no analogous retail process, product or service is feasible.

4.1.3 Enforcement Measurement Compliance means comparing performance levels provided to BellSouth retail customers with performance levels provided by BellSouth to the CLEC .

4.1.4 Test Statistic and Balancing Critical Value is the means by which enforcement will be determined using statistically valid equations. The Test Statistic and Balancing Critical Value are set forth in Exhibit D located on the Performance Measurements Reports website (labeled Appendix D attached), incorporated herein by this reference.

4.1.5 Cell is a grouping of transactions at which like-to-like comparisons are made. For example, all BellSouth retail POTS services, for residential customers, requiring a dispatch in a particular wire center, at a particular point in time will be compared directly to the CLEC resold services for residential customers, requiring a dispatch, in the same wire center, at a particular point in time. When determining compliance, these cells can have a positive or negative Test Statistic. See Exhibit C located on the Performance Measurements Reports website (labeled Appendix C attached), incorporated herein by this reference.

4.1.6 Affected Volume means that proportion of the total impacted individual CLEC volume or CLEC Aggregate volume for which remedies will be paid.

4.1.7 Parity Gap refers to the incremental departure from a compliant-level of service. This is also referred to as "diff" in the Statistical paper located at Exhibit C located on the Performance Measurements Reports website (labeled Appendix C attached), incorporated herein by this reference.

4.1.8 Tier-1 Enforcement Mechanisms means self-executing liquidated damages paid directly to the Treasury of the United States when BellSouth delivers non-compliant performance of any one of the Tier-1 Enforcement Measurement Elements for any month as calculated by BellSouth for an individual CLEC.

4.1.9 Tier-2 Enforcement Mechanisms means Assessments paid directly to the Treasury of the United States. Tier 2 Enforcement Mechanisms are triggered by three consecutive monthly failures in which BellSouth performance is out of compliance or does not meet the benchmarks for the aggregate of all CLEC data as calculated by BellSouth for a particular Tier-2 Enforcement Measurement Element.

4.1.10 Force Majeure

In the event performance of this Plan or the unduly services subject to the Plan, or any obligation hereunder, is either directly or indirectly prevented, restricted, or interfered with by reason of fire, flood, earthquake or like acts of God, wars, revolution, civil commotion, explosion, acts of public enemy, embargo, acts of the government in its sovereign capacity, labor difficulties, including without limitation, strikes, slowdowns, picketing, or boycotts, unavailability of equipment from vendor, changes requested by the CLEC, or any other circumstances beyond the reasonable control and without the fault or negligence of BellSouth, BellSouth shall be excused from such performance on a day-to-day basis to the extent of such prevention, restriction, or interference (and the other Party shall likewise be excused from performance of its obligations on a day-to-day basis until the delay, restriction or interference has ceased); provided however, that BellSouth shall use diligent efforts to avoid or remove such causes of non-performance and both Parties shall proceed whenever such causes are removed or cease.

4.2 Application

4.2.1 The Enforcement Mechanisms set forth in this section will commence only after BellSouth exercises an FCC grant of interLATA authority under section 271 of the Act within the State.

4.2.2 The application of the Tier-1 and Tier-2 Enforcement Mechanisms does not foreclose other legal and regulatory claims and remedies available to the CLEC.

4.2.3 Payment of any Tier-1 or Tier-2 Enforcement Mechanisms shall not be considered as an admission against interest or an admission of liability or culpability in any legal, regulatory or other proceeding relating to BellSouth's performance. The payment of any Tier-1 Enforcement Mechanisms shall be credited against any liability associated with or related to BellSouth's service performance.

4.2.4 It is not the intent of the Parties that BellSouth be liable for both Tier-2 Enforcement Mechanisms and any other assessments or sanctions imposed by the Commission. Nothing herein precludes BellSouth from seeking to set off Tier-2 Enforcement Mechanisms from any additional assessments imposed by the Commission for the same Enforcement Measure Element

4.2.5 The Commission acknowledges and agrees that the Enforcement Mechanisms contained in this attachment have been provided by BellSouth on a completely voluntary basis in order to maintain compliance between BellSouth and the CLEC Aggregate.

4.3 Methodology

4.3.1 Tier-1 Enforcement Mechanisms will be triggered by BellSouth's failure to achieve applicable Enforcement Measurement Compliance or Enforcement Measurement Benchmarks for an individual CLEC for the relevant State for a given Enforcement Measurement Element in a given month. Enforcement Measurement Compliance is based upon a Test Statistic and Balancing Critical Value calculated by BellSouth utilizing BellSouth generated data. The method of calculation is set forth in Exhibit D located on the Performance Measurements Reports website (labeled Appendix D attached), incorporated herein by this reference.

4.3.1.1 Tier-1 Enforcement Mechanisms apply on a per transaction basis for each negative cell and will escalate based upon the number of consecutive months that BellSouth has reported non-compliance.

4.3.1.2 Fee Schedule for Tier-1 Enforcement Mechanisms is shown on the Performance Measurement Reports website in Table-1 of Exhibit A (labeled Appendix A attached), incorporated herein by this reference. Failures beyond Month 6 will be subject to Month 6 fees.

4.3.2 Tier-2 Enforcement Mechanisms will be triggered by BellSouth's failure to achieve applicable Enforcement Measurement Compliance or Enforcement Measurement Benchmarks for the State for given Enforcement Measurement Elements for three consecutive months based upon a statistically valid equation calculated by BellSouth utilizing BellSouth generated data. The method of calculation is set forth in Exhibit D located on the Performance Measurements Reports website (labeled Appendix D attached), incorporated herein by this reference.

4.3.2.1 Tier- 2 Enforcement Mechanisms apply, for an aggregate of all CLEC data generated by BellSouth, on a per transaction basis for each negative cell for a particular Enforcement Measurement Element.

4.3.2.2 Fee Schedule for Total Quarterly Tier-2 Enforcement Mechanisms is shown on the Performance Measurement Reports website in

Table-2 of Exhibit A (labeled Appendix A attached), incorporated herein by this reference.

4.4 Payment of Tier-1 and Tier-2 Amounts

4.4.1 If BellSouth performance triggers an obligation to pay Tier-1 Enforcement Mechanisms, or an obligation to remit Tier-2 Enforcement Mechanisms to the Commission, BellSouth shall make payment in the required amount on the day upon which the final validated SEEM reports are posted on the Performance Measurements Reports website as set forth in Section 2.4 above.

4.4.2 If the CLEC disputes the amount paid for Tier-1 Enforcement Mechanisms, the CLEC shall submit a written claim to BellSouth within sixty-days (60) after the date of the performance measurement report for which the obligation arose. BellSouth shall investigate all claims and provide the CLEC written findings within thirty-days (30) after receipt of the claim. If BellSouth determines that additional amounts are owed, BellSouth shall paysuch additional amounts within thirty-days (30) after its findings along with 6% simple interest per annum.

4.4.3 At the end of each calendar year, BellSouth will have its independent auditing and accounting firm certify that the results of all Tier-1 and Tier-2 Enforcement Mechanisms were paid and accounted for in accordance with Generally Accepted Account Principles (GAAP).

4.5 Limitations of Liability

4.5.1 BellSouth will not be responsible for the CLEC's acts or omissions that cause performance measures to be missed or fail, including, but not limited to, accumulation and submission of orders at unreasonable quantities or times or failure to submit accurate orders or inquiries. BellSouth shall provide the CLEC with reasonable notice of such acts or omissions and provide the CLEC any such supporting documentation.

4.5.2 BellSouth shall not be obligated for Tier-1 or Tier-2 Enforcement Mechanisms for non-compliance with a performance measure if such non-compliance was the result of an act or omission by the CLEC that is in bad faith.

4.5.3 BellSouth shall not be obligated to pay Tier-1 Enforcement Mechanisms or Tier-2 Enforcement Mechanism for non-compliance with a performance measurement if such non-compliance was the result of any of the following: a Force Majeure event as defined in Section

4.1.10; an act or omission by the CLEC that is contrary to any of its obligations under its Interconnection Agreement with BellSouth; an act or omission by the CLEC that is contrary to any of its obligations under the Act, Commission rule, or state law; an act or omission associated with third-party systems or equipment.

4.6 Enforcement Mechanism Cap

4.6.1 BellSouth's total liability for the payment of Tier-1 and Tier-2 Enforcement Mechanisms shall be collectively capped at 36% of net revenue per year for the relevant State.

4.6.2 If projected payments exceed the cap, a proportional payment will be made to the respective parties.

4.6.3 If BellSouth's payment of Tier-1 and Tier-2 Enforcement Mechanisms would have exceeded the cap referenced in this section, the CLECs may commence a proceeding with the Commission to demonstrate why BellSouth should pay any amount in excess of the cap. The CLEC shall have the burden of proof to demonstrate why, under the circumstances, BellSouth should have additional liability.

4.7 Dispute Resolution

4.7.1 Notwithstanding any other provision of this document, any dispute regarding BellSouth's performance or obligations pursuant to this Plan shall be resolved by the Commission.

APPENDIX A

Fee Schedule

Table-1
LIQUIDATED DAMAGES TABLE FOR TIER-1 MEASURES

PER AFFECTED ITEM						
	Month 1	Month 2	Month3	Month4	Month 5	Month 6
Ordering	$40	$50	$60	$70	$80	$90
Provisioning	$100	$125	$175	$250	$325	$500
Provisioning UNE	$400	$450	$500	$550	$650	$800
Maintenance and Repair	$100	$125	$175	$250	$325	$500
Maintenance and Repair UNE	$400	$450	$500	$550	$650	$800

Table-2
REMEDY PAYMENTS FOR TIER-2 MEASURES

	Per Affected Item
OSS Pre-Ordering	$20
Ordering	$60
Provisioning	$300
UNE Provisioning	$875
Maintenance and Repair	$300
UNE Maintenance and Repair	$875
Service Order Accuracy	$50

APPENDIX B

SEEM Sub-Metrics

SEEM TIER-1 SUB-METRICS

1. Percent Provisioning Troubles in 5 days of Service Order Completion – Resale Residence and Business
2. Percent Provisioning Troubles in 5 days of Service Order Completion – UNE Analog Loops
3. Percent Provisioning Troubles in 5 days of Service Order Completion – UNE Digital Loops (Including xDSL)
4. Percent Provisioning Troubles in 5 days of Service Order Completion – UNE Loop and Port Combinations (UNEP)
5. Percent Provisioning Troubles in 5 days of Service Order Completion – EELS (Including NSCs)
6. Customer Trouble Report Rate – Resale Residence and Business
7. Customer Trouble Report Rate Completion – UNE Analog Loops
8. Customer Trouble Report Rate – UNE Digital Loops (Including xDSL)
9. Customer Trouble Report Rate – UNE Loop and Port Combinations (UNEP)
10. Customer Trouble Report Rate – EELS (Including NSCs)
11. Percent Repeat Troubles in 30 days – Resale Residence and Business
12. Percent Repeat Troubles in 30 days – UNE Analog Loops
13. Percent Repeat Troubles in 30 days – UNE Digital Loops (Including xDSL)
14. Percent Repeat Troubles in 30 days – UNE Loop and Port Combinations (UNEP)
15. Percent Repeat Troubles in 30 days – EELS (Including NSCs)
16. Maintenance Average Duration – Resale Residence and Business
17. Maintenance Average Duration – UNE Analog Loops
18. Maintenance Average Duration – UNE Digital Loops (Including xDSL)
19. Maintenance Average Duration – UNE Loop and Port Combinations (UNEP)
20. Maintenance Average Duration – EELS (Including NSCs)
21. Percent Missed Installation – Resale Residence and Business
22. Percent Missed Installation – UNE Analog Loops
23. Percent Missed Installation – UNE Digital Loops (Including xDSL)
24. Percent Missed Installation – UNE Loop and Port Combinations (UNEP)
25. Percent Missed Installation – EELS (Including NSCs)

Tier-2 Submetrics

1. Average Response Time and Response Interval
2. Firm Order Confirmation – Mechanized
3. Firm Order Confirmation – Non-Mechanized
4. Reject Interval –Mechanized
5. Reject Interval – Non-Mechanized
6. Percent Provisioning Troubles in 5 days of Service Order Completion – Resale Residence and Business
7. Percent Provisioning Troubles in 5 days of Service Order Completion – UNE Analog Loops
8. Percent Provisioning Troubles in 5 days of Service Order Completion – UNE Digital Loops (Including xDSL)
9. Percent Provisioning Troubles in 5 days of Service Order Completion – UNE Loop and Port Combinations (UNEP)
10. Percent Provisioning Troubles in 5 days of Service Order Completion – EELS (Including NSCs)
11. Customer Trouble Report Rate – Resale Residence and Business
12. Customer Trouble Report Rate Completion – UNE Analog Loops
13. Customer Trouble Report Rate – UNE Digital Loops (Including xDSL)
14. Customer Trouble Report Rate – UNE Loop and Port Combinations (UNEP)
15. Customer Trouble Report Rate – EELS (Including NSCs)
16. Percent Repeat Troubles in 30 days – Resale Residence and Business
17. Percent Repeat Troubles in 30 days – UNE Analog Loops
18. Percent Repeat Troubles in 30 days – UNE Digital Loops (Including xDSL)
19. Percent Repeat Troubles in 30 days – UNE Loop and Port Combinations (UNEP)
20. Percent Repeat Troubles in 30 days – EELS (Including NSCs)
21. Maintenance Average Duration – Resale Residence and Business
22. Maintenance Average Duration – UNE Analog Loops
23. Maintenance Average Duration – UNE Digital Loops (Including xDSL)
24. Maintenance Average Duration – UNE Loop and Port Combinations (UNEP)
25. Maintenance Average Duration – EELS (Including NSCs)
26. Service Order Accuracy – Resale
27. Service Order Accuracy – UNE Loop and Port Combinations (UNEP)
28. Service Order Accuracy – UNE Loop
29. Percent Missed Installation – Resale Residence and Business
30. Percent Missed Installation – UNE Analog Loops
31. Percent Missed Installation – UNE Digital Loops (Including xDSL)
32. Percent Missed Installation – UNE Loop and Port Combinations (UNEP)
33. Percent Missed Installation – EELS (Including NSCs)

APPENDIX C

Statistical Methodology

Statistical Methods for BellSouth Performance Measure Analysis

I. Necessary Properties for a Test Methodology

The statistical process for testing if competing local exchange carriers (CLECs) customers are being treat equally with BellSouth (BST) customers involves more than just a mathematical formula. Three key elements need to be considered before an appropriate decision process can be developed. These are

- the type of data,
- the type of comparison, and
- the type of performance measure.

Once these elements are determined a test methodology should be developed that complies with the following properties.

- Like-to-Like Comparisons. When possible, data should be compared at appropriate levels, e.g. wire center, time of month, dispatched, and residential, new orders. The testing process should:
  - Identify variables that may affect the performance measure.
  - Record these important confounding covariates.
  - Adjust for the observed covariates in order to remove potential biases and to make the CLEC and the ILEC units as comparable as possible.
- Aggregate Level Test Statistic. Each performance measure of interest should be summarized by one overall test statistic giving the decision maker a rule that determines whether a statistically significant difference exists. The test statistic should have the following properties.
  - The method should provide a single overall index, on a standard scale.
  - If entries in comparison cells are exactly proportional over a covariate, the aggregated index should be very nearly the same as if comparisons on the covariate had not been done.
  - The contribution of each comparison cell should depend on the number of observations in the cell.
  - Cancellation between comparison cells should be limited.
  - The index should be a continuous function of the observations.

- Production Mode Process. The decision system must be developed so that it does not require intermediate manual intervention, i.e. the process must be mechanized to the extent possible.
  - Calculations are well defined for possible eventualities.
  - The decision process is an algorithm that needs no manual intervention.
  - Results should be arrived at in a timely manner.
  - The system must recognize that resources are needed for other performance measure-related processes that also must be run in a timely manner.
  - The system should be auditable, and adjustable over time.
- Balancing. The testing methodology should balance Type I and Type II Error probabilities.
  - P(Type I Error) = P(Type II Error) for well defined null and alternative hypotheses.
  - The formula for a test's balancing critical value should be simple enough to calculate using standard mathematical functions, i.e. one should avoid methods that require computationally intensive techniques.
  - Little to no information beyond the null hypothesis, the alternative hypothesis, and the number of observations should be required for calculating the balancing critical value.

- Trimming. Trimming of extreme observations from BellSouth and CLEC distributions is needed in order to ensure that a fair comparison is made between performance measures. Three conditions are needed to accomplish this goal. These are:
  - Trimming should be based on a general rule that can be used in a production setting.
  - Trimmed observations should not simply be discarded; they need to be examined and possibly used in the final decision making process.
  - Trimming should only be used on performance measures that are sensitive to "outliers."

Measurement Types

The performance measures that will undergo testing are of four types:

1) means
2) proportions,
3) rates, and
4) ratio

While all four have similar characteristics, proportions and rates are derived from count data while means and ratios are derived from interval measurements.

II. Testing Methodology – The Truncated Z

Many covariates are chosen in order to provide deep comparison levels. In each comparison cell, a Z statistic is calculated. The form of the Z statistic may vary depending on the performance measure, but it should be distributed approximately as a standard normal, with mean zero and variance equal to one. Assuming that the test statistic is derived so that it is negative when the performance for the CLEC is worse than for the ILEC, a positive truncation is done – i.e. if the result is negative it is left alone, if the result is positive it is changed to zero. A weighted average of the truncated statistics is calculated where a cell weight depends on the volume of BST and CLEC orders in the cell. The weighted average is re-centered by the theoretical mean of a truncated distribution, and this is divided by the standard error of the weighted average. The standard error is computed assuming a fixed effects model.

Proportion Measures

For performance measures that are calculated as a proportion, in each adjustment cell, the truncated Z and the moments for the truncated Z can be calculated in a direct manner. In adjustment cells where proportions are not close to zero or one, and where the sample sizes are reasonably large, a normal approximation can be used. In this case, the moments for the truncated Z come directly from properties of the standard normal distribution. If the normal approximation is not appropriate, then the Z statistic is calculated from the hypergeometric distribution. In this case, the moments of the truncated Z are calculated exactly using the hypergeometric probabilities.

Rate Measures

The truncated Z methodology for rate measures has the same general structure for calculating the Z in each cell as proportion measures. For a rate measure, there are a fixed number of circuits or units for the CLEC, n_{2j} and a fixed number of units for BST, n_{1j}. Suppose that the performance measure is a "trouble rate." The modeling assumption is that the occurrence of a trouble is independent between units and the number of troubles in n circuits follows a Poisson distribution with mean λ n where λ is the probability of a trouble in 1 circuit and n is the number of circuits.

In an adjustment cell, if the number of CLEC troubles is greater than 15 and the number of BST troubles is greater than 15, then the Z test is calculated using the normal approximation to the Poisson. In this case, the moments of the truncated Z come directly from properties of the standard normal distribution. Otherwise, if there are very few troubles, the number of CLEC troubles can be modeled using a binomial distribution with n equal to the total number of troubles (CLEC plus BST

troubles.) In this case, the moments for the truncated Z are calculated explicitly using the binomial distribution.

Mean Measures

For mean measures, an adjusted t statistic is calculated for each like-to-like cell which has at least 7 BST and 7 CLEC transactions. A permutation test is used when one or both of the BST and CLEC sample sizes is less than 6. Both the adjusted t statistic and the permutation calculation are described in the technical appendix.

Ratio Measures

Rules will be given for computing a cell test statistic for a ratio measure, however, the current plan for measures in this category, namely billing accuracy, does not call for the use of a Z parity statistic.

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APPENDIX D

Technical Description

We start by assuming that any necessary trimming[1] of the data is complete, and that the data are disaggregated so that comparisons are made within appropriate classes or adjustment cells that define "like" observations.

Notation and Exact Testing Distributions

Below, we have detailed the basic notation for the construction of the truncated z statistic. In what follows the word "cell" should be taken to mean a like-to-like comparison cell that has both one (or more) ILEC observation and one (or more) CLEC observation.

L = the total number of occupied cells

j = 1,…,L; an index for the cells

n_{1j} = the number of ILEC transactions in cell j

n_{2j} = the number of CLEC transactions in cell j

n_j = the total number transactions in cell j; $n_{1j}+ n_{2j}$

X_{1jk} = individual ILEC transactions in cell j; $k = 1,\ldots, n_{1j}$

X_{2jk} = individual CLEC transactions in cell j; $k = 1,\ldots, n_{2j}$

Y_{jk} = individual transaction (both ILEC and CLEC) in cell j

$$= \begin{cases} X_{1jk} & k = 1,\ldots,n_{1j} \\ X_{2jk} & k = n_{1j}+1,\ldots,n_j \end{cases}$$

$\Phi^{-1}(\cdot)$ = the inverse of the cumulative standard normal distribution function

For Mean Performance Measures the following additional notation is needed.

$\overline{X}_{1j}$ = The ILEC sample mean of cell j

$\overline{X}_{2j}$ = The CLEC sample mean of cell j

s_{1j}^2 = The ILEC sample variance in cell j

s_{2j}^2 = The CLEC sample variance in cell j

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[1] When it is determined that a measure should be trimmed, a trimming rule that is easy to implement in a production setting is:

> **Trim the ILEC observations to the largest CLEC value from all CLEC observations in the month under consideration.**

That is, no CLEC values are removed; all ILEC observations greater than the largest CLEC observation are trimmed.

$\{y_{jk}\}$ = a random sample of size n_{2j} from the set of $Y_{j1},\ldots,Y_{jn_j}$; $k = 1,\ldots,n_{2j}$

M_j = The total number of distinct pairs of samples of size n_{1j} and n_{2j};

$$= \binom{n_j}{n_{1j}}$$

The exact parity test is the permutation test based on the "modified Z" statistic. For large samples, we can avoid permutation calculations since this statistic will be normal (or Student's t) to a good approximation. For small samples, where we cannot avoid permutation calculations, we have found that the difference between "modified Z" and the textbook "pooled Z" is negligible. We therefore propose to use the permutation test based on pooled Z for small samples. This decision speeds up the permutation computations considerably, because for each permutation we need only compute the sum of the CLEC sample values, and not the pooled statistic itself.

A permutation probability mass function distribution for cell j, based on the "pooled Z" can be written as

$$PM(t) = P(\sum_k y_{jk} = t) = \frac{\textit{the number of samples that sum to } t}{M_j},$$

and the corresponding cumulative permutation distribution is

$$CPM(t) = P(\sum_k y_{jk} \le t) = \frac{\textit{the number of samples with sum } \le t}{M_j}.$$

For Proportion Performance Measures the following notation is defined

a_{1j} = The number of ILEC cases possessing an attribute of interest in cell j

a_{2j} = The number of CLEC cases possessing an attribute of interest in cell j

a_j = The number of cases possessing an attribute of interest in cell j; $a_{1j}+ a_{2j}$

The exact distribution for a parity test is the hypergeometric distribution. The hypergeometric probability mass function distribution for cell j is

$$HG(h) = P(H = h) = \begin{cases} \dfrac{\binom{n_{1j}}{h}\binom{n_{2j}}{a_j - h}}{\binom{n_j}{a_j}}, & \max(0, a_j - n_{2j}) \le h \le \min(a_j, n_{1j}) \\ 0 & \text{otherwise} \end{cases},$$

and the cumulative hypergeometric distribution is

$$CHG(x) = P(H \le x) = \begin{cases} 0 & x < \max(0, a_j - n_{2j}) \\ \sum_{h=\max(0,a_j-n_{1j})}^{x} HG(h), & \max(0, a_j - n_{2j}) \le x \le \min(a_j, n_{1j}) \\ 1 & x > \min(a_j, n_{1j}) \end{cases}.$$

For Rate Measures, the notation needed is defined as

b_{1j} = The number of ILEC base elements in cell j

b_{2j} = The number of CLEC base elements in cell j

b_j = The total number of base elements in cell j; $b_{1j} + b_{2j}$

$\hat{r}_{1j}$ = The ILEC sample rate of cell j; n_{1j}/b_{1j}

$\hat{r}_{2j}$ = The CLEC sample rate of cell j; n_{2j}/b_{2j}

q_j = The relative proportion of ILEC elements for cell j; b_{1j}/b_j

The exact distribution for a parity test is the binomial distribution. The binomial probability mass function distribution for cell j is

$$BN(k) = P(B = k) = \begin{cases} \binom{n_j}{k} q_j^k (1-q_j)^{n_j-k}, & 0 \le k \le n_j \\ 0 & \text{otherwise} \end{cases},$$

and the cumulative binomial distribution is

$$CBN(x) = P(B \le x) = \begin{cases} 0 & x < 0 \\ \sum_{k=0}^{x} BN(k), & 0 \le x \le n_j \\ 1 & x > n_j \end{cases}.$$

For Ratio Performance Measures the following additional notation is needed.

U_{1jk} = additional quantity of interest of an individual ILEC transaction in cell j; k = $1,\ldots, n_{1j}$

U_{2jk} = additional quantity of interest of an individual CLEC transaction in cell j; k = $1,\ldots, n_{2j}$

$\hat{R}_{ij}$ = the ILEC (I = 1) or CLEC (i = 2) ratio of the total additional quantity of interest to the base transaction total in cell j, i.e., $\sum_k U_{ijk} \Big/ \sum_k X_{ijk}$

Calculating the Truncated Z

The general methodology for calculating an aggregate level test statistic is outlined below.

1. **Calculate cell weights**, W_j. A weight based on the number of transactions is used so that a cell, which has a larger number of transactions, has a larger weight. The actual weight formulae will depend on the type of measure.

Mean or Ratio Measure

$$W_j = \sqrt{\frac{n_{1j} n_{2j}}{n_j}}$$

Proportion Measure

$$W_j = \sqrt{\frac{n_{2j} n_{1j}}{n_j} \cdot \frac{a_j}{n_j} \cdot \left(1 - \frac{a_j}{n_j}\right)}$$

Rate Measure

$$W_j = \sqrt{\frac{b_{1j} b_{2j}}{b_j} \cdot \frac{n_j}{b_j}}$$

2. **In each cell, calculate a Z value**, Z_j. A Z statistic with mean 0 and variance 1 is needed for each cell.

   - If $W_j = 0$, set $Z_j = 0$.
   - Otherwise, the actual Z statistic calculation depends on the type of performance measure.

Mean Measure

$$Z_j = \Phi^{-1}(\alpha)$$

where α is determine by the following algorithm.

If $\min(n_{1j}, n_{2j}) > 6$, then determine α as

$$\alpha = P(t_{n_{1j}-1} \le T_j),$$

that is, α is the probability that a t random variable with $n_{1j} - 1$ degrees of freedom, is less than

$$T_j = \begin{cases} t_j + \frac{g}{6}\left(\frac{n_{1j}+2n_{2j}}{\sqrt{n_{1j}\, n_{2j}(n_{1j}+n_{2j})}}\right)\left(t_j^2 + \frac{n_{2j}-n_{1j}}{n_{1j}+2n_{2j}}\right) & t_j \ge t_{\min j} \\ t_j + \frac{g}{6}\left(\frac{n_{1j}+2n_{2j}}{\sqrt{n_{1j}\, n_{2j}(n_{1j}+n_{2j})}}\right)\left(t_{\min j}^2 + \frac{n_{2j}-n_{1j}}{n_{1j}+2n_{2j}}\right) & \text{otherwise} \end{cases},$$

where

$$t_j = \frac{\bar{X}_{1j} - \bar{X}_{2j}}{s_{1j}\sqrt{\frac{1}{n_{1j}} + \frac{1}{n_{2j}}}},$$

$$t_{\min j} = \frac{-3\sqrt{n_{1j}n_{2j}n_j}}{g(n_{1j}+2n_{2j})}$$

and g is the median value of all values of

$$\gamma_{1j} = \frac{n_{1j}}{(n_{1j}-1)(n_{1j}-2)}\sum_k\left(\frac{X_{1jk} - \bar{X}_{1j}}{s_{1j}}\right)^3$$

with $n_{1j} > n_{3q}$ for all values of j. n_{3q} is the 3 quartile of all values of n_{1j}.

Note, that t_j is the "modified Z" statistic. The statistic T_j is a "modified Z" corrected for the skewness of the ILEC data.

If $\min(n_{1j}, n_{2j}) \leq 6$, and

a) $M_j \leq 1{,}000$ (the total number of distinct pairs of samples of size n_{1j} and n_{2j} is 1,000 or less).

- Calculate the sample sum for all possible samples of size n_{2j}.
- Rank the sample sums from smallest to largest. Ties are dealt by using average ranks.
- Let R_0 be the rank of the observed sample sum with respect all the sample sums.

$$\alpha = 1 - \frac{R_0 - 0.5}{M_j}$$

b) $M_j > 1{,}000$

- Draw a random sample of 1,000 sample sums from the permutation distribution.
- Add the observed sample sum to the list. There are a total of 1001 sample sums. Rank the sample sums from smallest to largest. Ties are dealt by using average ranks.
- Let R_0 be the rank of the observed sample sum with respect all the sample sums.

$$\alpha = 1 - \frac{R_0 - 0.5}{1001}.$$

Proportion Measure

$$Z_j = \frac{n_j\, a_{1j} - n_{1j}\, a_j}{\sqrt{\frac{n_{1j}\, n_{2j}\, a_j\, (n_j - a_j)}{n_j - 1}}}.$$

Rate Measure

$$Z_j = \frac{n_{1j} - n_j\, q_j}{\sqrt{n_j\, q_j (1 - q_j)}}$$

Ratio Measure

$$Z_j = \frac{\hat{R}_{1j} - \hat{R}_{2j}}{\sqrt{V(\hat{R}_{1j})\left(\frac{1}{n_{1j}} + \frac{1}{n_{2j}}\right)}}$$

$$V(\hat{R}_{1j}) = \frac{\sum_k \left(U_{1jk} - \hat{R}_{1j}X_{1jk}\right)^2}{\bar{X}_{1j}^2(n_{1j}-1)} = \frac{\sum_k U_{1jk}^2 - 2\hat{R}_{1j}\sum_k \left(U_{1jk}X_{1jk}\right) + \hat{R}_{1j}^2\sum_k X_{1jk}^2}{\bar{X}_{1j}^2(n_{1j}-1)}$$

3. **Obtain a truncated Z value for each cell,** Z_j^*. To limit the amount of cancellation that takes place between cell results during aggregation, cells whose results suggest possible favoritism are left alone. Otherwise the cell statistic is set to zero. This means that positive equivalent Z values are set to 0, and negative values are left alone. Mathematically, this is written as

$$Z_j^* = \min(0, Z_j).$$

4. **Calculate the theoretical mean and variance of the truncated statistic under the null hypothesis of parity,** $E(Z_j^* | H_0)$ and $Var(Z_j^* | H_0)$. In order to compensate for the truncation in step 3, an aggregated, weighted sum of the Z_j^* will need to be centered and scaled properly so that the final aggregate statistic follows a standard normal distribution.

   - If $W_j = 0$, then no evidence of favoritism is contained in the cell. The formulae for calculating $E(Z_j^* | H_0)$ and $Var(Z_j^* | H_0)$ cannot be used. Set both equal to 0.

   - If $\min(n_{1j}, n_{2j}) > 6$ for a mean measure, $\min\left\{a_{1j}\left(1-\frac{a_{1j}}{n_{1j}}\right), a_{2j}\left(1-\frac{a_{2j}}{n_{2j}}\right)\right\} > 9$ for a proportion measure, $\min\left(n_{1j}, n_{2j}\right) > 15$ and $n_j q_j (1-q_j) > 9$ for a rate measure, or n_{1j} and n_{2j} are large for a ratio measure then

$$E(Z_j^* | H_0) = -\frac{1}{\sqrt{2\pi}}, \text{ and}$$

$$Var(Z_j^* | H_0) = \frac{1}{2} - \frac{1}{2\pi}.$$

   - Otherwise, determine the total number of values for Z_j^*. Let z_{ji} and θ_{ji}, denote the values of Z_j^* and the probabilities of observing each value, respectively.

$$E(Z_j^* \mid H_0) = \sum_i \theta_{ji} z_{ji} \text{ ,and}$$

$$Var(Z_j^* \mid H_0) = \sum_i \theta_{ji} z_{ji}^2 - \left[E(Z_j^* \mid H_0) \right]^2 .$$

The actual values of the z's and θ's depends on the type of measure.

Mean Measure

$$N_j = \min(M_j, 1{,}000), \quad i = 1, \ldots, N_j$$

$$z_{ji} = \min\left\{0, \Phi^{-1}\left(1 - \frac{R_i - 0.5}{N_j}\right)\right\} \quad \text{where } R_i \text{ is the rank of sample sum i}$$

$$\theta_j = \frac{1}{N_j}$$

Proportion Measure

$$z_{ji} = \min\left\{0, \frac{n_j\, i - n_{1j}\, a_j}{\sqrt{\frac{n_{1j}\, n_{2j}\, a_j\, (n_j - a_j)}{n_j - 1}}}\right\}, \quad i = \max(0, a_j - n_{2j}), \ldots, \min(a_j, n_{1j})$$

$$\theta_{ji} = HG(i)$$

Rate Measure

$$z_{ji} = \min\left\{0, \frac{i - n_j\, q_j}{\sqrt{n_j\, q_j (1 - q_j)}}\right\}, \quad i = 0, \ldots, n_j$$

$$\theta_{ji} = BN(i)$$

Ratio Measure

The performance measure that is in this class is billing accuracy. If a parity test were used, the sample sizes for this measure are quite large, so there is no need for a small sample technique. If one does need a small sample technique, then a re-sampling method can be used.